RAMOO 2021

Workshop on Recent Advances in Multi-Objective Optimization

September 23rd, 2021

University of Wuppertal, Germany – Online

Organized by Michael Stiglmayr, Kathrin Klamroth & Britta Schulze

Kindly supported by

DFG Deutsche Forschungsgemeinschaft

BERGISCHE UNIVERSITÄT WUPPERTAL

Thursday, September 23rd, 2021

Workshop Program, RAMOO 2021

08:45 – 09:00 (CEST)	**Welcome**, Michael Stiglmayr
09:00 – 10:00	**Keynote 1**, Chair: Xavier Gandibleux Sophie Parragh: *Towards multi-objective mixed integer linear programming*
10:00 – 11:00	**Session 1**, Chair: Daniel Vanderpooten Kathrin Prinz: *The weighted p-norm weight set decomposition for multiobjective discrete optimization problems* Ahmet Yükseltürk: *Weight set characterization of Chebyshev distance scalarization method for multiobjective integer optimization problem*
11:00 – 11:30	**Coffee Break in Gather.town**
11:30 – 13:00	**Session 2**, Chair: Firdevs Ulus Julia Sudhoff: *Bi-objective optimization with ordinal cost coefficients: The example of matroid problems* Lavinia Amorosi: *A new enumerative recursive procedure for the bi-objective integer minimum cost flow problem* Özlem Karsu: *An objective space based algorithm for biobjective mixed integer programming problems*
13:00 – 14:00	**Lunch Break in Gather.town**
14:00 – 15:00	**Session 3**, Chair: Kathrin Klamroth Leo Warnow: *Computing an enclosure for multi-objective mixed-integer convex optimization problems* Manuel Berkemeier: *A derivative-free local optimizer for multi-objective blackbox problems*
15:00 – 15:30	**Coffee Break in Gather.town**
15:30 – 16:30	**Session 4**, Chair: Fritz Bökler Marianna De Santis: *Detecting the efficient set of multiobjective integer quadratic programming problems* Fabian Chlumsky-Harttmann: *Cutting planes in robust multi-objective optimization*
16:30 – 17:30	**Keynote 2**, Chair: Stefan Ruzika Luís Paquete: *Hypervolume-based Optimization: Results and Challenges*
17:30	**Closing & Group Picture**, Michael Stiglmayr, Announcing RAMOO 2022

Cover Photo: Ralf Baumgarten

Abstracts

Keynote Talks

Sophie Parragh (Johannes Kepler University Linz)

Towards multi-objective mixed integer linear programming

In the single objective domain, general purpose mixed integer linear programming solvers have become indispensable tools for either solving the underlying optimization problems directly or as major building blocks for heuristics. While it has been acknowledged that many (if not all) practical optimization problems feature more than one objective, general purpose exact methods for solving any multi-objective mixed integer linear programming problem are still under development. In this talk, recent advances in important ingredients to such methods which rely on the branch-and-bound idea are presented. These ingredients range from bound (set) computation schemes to branching rules, node selection strategies, cutting plane generation and primal heuristics. The focus is put on those ideas which exploit the multi-objective nature of the underlying optimization problem and their potential advantages and disadvantages are highlighted. Furthermore, open challenges which include, e.g., stability issues, test instance design, and meaningful gap measures are discussed.

Luís Paquete (University of Coimbra)

Hypervolume-based optimization: results and challenges

The hypervolume indicator measures the multidimensional volume of the union of axis-parallel boxes, each of which spanned by a nondominated point and a pre-defined reference point. This indicator has shown to have interesting properties, and has gained popularity as a performance assessment method, as a selection criterion, and as an archiving strategy for multiobjective evolutionary algorithms. Moreover, under appropriate assumptions about the location of the reference point in the objective space, the hypervolume indicator takes its maximum value at the nondominated set. This result suggests that optimizing the hypervolume indicator might also be useful in the context of exact approaches for multiobjective optimization problems.
In this talk, we consider two possibilities of applying the hypervolume indicator in the context of multiobjective combinatorial optimization: i) to consider the hypervolume indicator from the perspective of representation quality, where the goal is, given a nondominated set, to find a subset of a given cardinality that maximizes the hypervolume indicator, and ii) to use the hypervolume indicator as a scalarization method, leading to search procedures that find the nondominated set, or a subset of it. We discuss applications of these methods to particular biobjective combinatorial optimization problems as well as challenges that arise when considering more than two objectives.

Invited Talks (in alphabetical order)

Lavinia Amorosi, Matthias Ehrgott, and Benjamin Weißing (Universita' di Roma – La Sapienza)

A new enumerative recursive procedure for the bi-objective integer minimum cost flow problem

In this talk we describe a new two-phase algorithm to generate a minimal complete set of efficient solutions for the bi-objective integer minimum cost flow (BIMCF) problem [1]. For the first phase, we propose the adoption of the dual variant of Benson's algorithm [2] taking advantage of the total unimodularity of the coefficient matrix of this problem. The second phase consists in a new enumerative recursive procedure, based on increasing values of reduced costs of variables in associated weighted linear programs [3]. This procedure is able to generate all integer feasible flows on a connected network. Combined with bounds for the costs of efficient flows, the two-phase method finds a minimal or maximal complete set of efficient solutions. The description of this procedure is accompanied by preliminary numerical tests show the effectiveness of the method and issues related to degeneracy.

[1] Raith, A., Ehrgott, M., A two-phase algorithm for the biobjective integer minimum cost flow problem. *Computers & Operations Research*, 36 (2009), 1945-1954.
[2] Hamel, A. Löhne, A., Rudloff, B., Benson type algorithms for linear vector optimization and applications, *Journal of Global Optimization*, 59 (2014) 811-836.
[3] Amorosi L., Bi-criteria network optimization: problems and algorithms, Ph.D. thesis, University of Rome, Sapienza (2018).

Manuel Berkemeier (University of Paderborn)

A derivative-free local optimizer for multi-objective blackbox problems

In real-world applications, multi-objective optimization problems might arise where some objectives are computationally expensive to evaluate, with no gradient information available. I will present some results with our derivative-free local optimizer aimed at such non-linear blackbox problems. It employs a trust-region strategy and local surrogate models (e.g., polynomials or radial basis function models) to save function evaluations. Besides talking about the theory I also want to touch on our Julia implementation and our plans for the future, concerning globalization approaches and the treatment of constraints.

Fabian Chlumsky-Harttmann (TU Kaiserslautern)

Cutting planes in robust multi-objective optimization

Real-world optimization problems often do not just involve multiple objectives but also a degree of uncertainty in some parameters – be it through prediction errors about parameters like demand or returns that will only be determined in the future or existing measurement errors. Robust multi-objective optimization takes this into account. Theory in this field has been developed in recent years and concepts generalizing efficiency to uncertain problems and robustness to multi-objective problems have been established. However, practicable solution methods are still rare. We propose a method that combines a cutting plane approach, that iteratively adds scenarios and thereby increases the uncertainty set well-known in robust optimization, with weighted-sum scalarization and – with a focus on linear biobjective problems – dichotomic search (Aneja-Nair method) specifically. We state different algorithms we have implemented all using cutting planes and weighted-sum, prove their correctness, discuss their respective strengths and weaknesses and compare their numerical performances.

Marianna De Santis (Universita' di Roma – La Sapienza)

Detecting the efficient set of multiobjective integer quadratic programming problems

Multiobjective integer optimization refers to mathematical programming problems where more than one objective function needs to be optimized simultaneously and all the variables are constrained to be integer. We present a branch-and-bound algorithm for minimizing multiple convex quadratic objective functions over integer variables. Our method looks for efficient points by fixing subsets of variables to integer values and by using lower bounds in the form of hyperplanes in the image space derived from the continuous relaxations of the restricted objective functions. We show that the algorithm stops after finitely many fixings of variables with detecting both the full efficient and the nondominated set of multiobjective strictly convex quadratic integer problems. A major advantage of the approach is that the expensive calculations are done in a preprocessing phase so that the nodes in the branch-and-bound tree can be enumerated fast. We show numerical experiments on biobjective instances and on instances with three and four objectives.

Özlem Karsu, Deniz Emre, Firdevs Ulus (Bilkent University)

An objective space based algorithm for biobjective mixed integer programming problems

We propose an objective space based exact solution algorithm for biobjective mixed integer programming problems. The algorithm solves scalarization models in order to explore predetermined regions of the objective space called boxes, defined by two nondominated points. At each iteration of the algorithm, a box is explored either by a weighted sum or a Pascoletti-Serafini scalarization to determine nondominated line segments and points. We demonstrate the applicability of the algorithm through computational experiments.

Kathrin Prinz (TU Kaiserslautern)

The weighted p-norm weight set decomposition for multiobjective discrete optimization problems

Scalarization is a frequently used approach that transforms a multiobjective problem into a single objective optimization problem. Two well-known scalarization techniques are the weighted sum and weighted Tchebycheff scalarization. Both can be interpreted as a p-norm scalarization: a positive weight is applied to each objective function, and the distance to a given reference point is minimized under a p-norm. Weight set decompositions have been thoroughly explored for the weighted sum and Tchebycheff scalarization of discrete optimization problems: for each image, the weight set component of this image contains all weight vectors for whose scalarization it is optimal. The resulting decomposition offers additional insight into the nondominated frontier and the construction forms an integral part of algorithms that seek to generate the nondominated frontier or approximations. In this talk, we generalize these results to arbitrary p-norm scalarizations. We first study how different p-norm scalarizations interrelate. Then, we introduce their weight set decompositions, provide fundamental geometrical properties and study their impact on the image set.

Julia Sudhoff, Kathrin Klamroth, Michael Stiglmayr (University of Wuppertal)

Bi-objective optimization with ordinal cost coefficients: the example of matroid problems

Bi-objective optimization problems on matroids are in general NP-hard and intractable, but if one of the objective functions is restricted to binary cost coefficients the problem becomes efficently solvable. A binary objective function often represents two categories and are thus a special case of ordinal coefficients that are in general non-additive. In this talk we consider ordinal objective functions with more than two categories. This leads to a new class of optimization problems, which we investigate especially with respect to their set of non-dominated outcome vectors. We present a transformation of an ordinal objective function to vector-valued objective functions with non-negative integer values. We show that all suggested models are efficiently solvable by a matroid intersection algorithm. Moreover, we discuss the application of this transformation to general combinatorial optimization problems with fixed cardinality and ordinal coefficients.

Leo Warnow (TU Ilmenau)

Computing an enclosure for multi-objective mixed-integer convex optimization problems

In multi-objective mixed-integer convex optimization multiple convex objective functions need to be optimized simultaneously while some of the variables are only allowed to take integer values. In this talk, we present a new approach to compute an enclosure of the nondominated set of such optimization problems. More precisely, we decompose the multi-objective mixed-integer convex optimization problem into several multi-objective continuous convex optimization problems, which we refer to as patches. We then dynamically compute and improve coverages of the nondominated sets of those patches to finally combine them to obtain an enclosure of the nondominated set of the multi-objective mixed-integer convex optimization problem. Additionally, we introduce a mechanism to reduce the number of patches that need to be considered in total.

<u>Ahmet Yükseltürk</u> and Serpil Sayın (Koç University)

Weight set characterization of Chebyshev distance scalarization method for multiobjective integer optimization problem

One of the methods to solve a multiobjective optimization problem (MOP) is to solve a sequence of single objective optimization problems called scalarizations. Minimizing weighted Chebyshev distance is one of the scalarization methods. For every nondominated solution of MOP, there is a weight, such that after solving the scalarization problem with this weight we obtain this nondominated solution. A multiobjective integer optimization problem (MOIP) with a bounded feasible region has finitely many nondominated solutions. As a consequence, we can associate more than one weight with a nondominated solution. In this setting we have weight sets of solutions. Specifically, the weight set is a collection of weights, such that scalarization problems formed with weights from this set will result in the same nondominated solution. The weight sets in weighted Chebyshev distance scalarization are known to be non-convex. However, we lack a thorough understanding of these weight sets. Results are available only for biobjective and triobjective MOIP models. Information about neighborhood properties of weight sets in general case are presented in decision making context. In contrast, there is a rich literature about the structure of weight sets in weighted sum scalarization and many algorithms make use of this information. Here we show that the weight set associated with every nondominated solution of MOIP with finitely many objectives is starshaped and contains as many affinely independent points as the number of objectives. When any pair of nondominated solutions do not share a component, weight sets partition the weight space. Otherwise, weight sets intersect. We think that this characterization may facilitate the development of a new class of algorithms to generate nondominated solutions of MOIP.